**Topics: Normal distribution, Functions of Random Variables**

1. **The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
2. **0.3875**
3. **0.2676**
4. **0.5**
5. **0.6987**

**Answer 01:**

We have a normal distribution with μ= 45 and σ = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50

P(X > 50).P(X > 50) = 1 -P(X ≤ 50).Z = (X -)/ = (X -45)/8.0

P(X ≤ 50) = P(Z ≤ (50 -45)/8.0) = P(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4

= 26.6% or 0.2676

1. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.**
2. **More employees at the processing center are older than 44 than between 38 and 44.**
3. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

Answer 02:

We have a normal distribution with ***μ*** = 38 and ***σ***  = 6.

Let X be the number of employees.

1. Probability of employees greater than age of 44 is

P(X>44) = 1 -P(X ≤ 44).Z = (X -38)/6

P(X ≤ 44) = P(Z ≤ (44 -38)/6) = P(Z ≤ 1) = 84.1345%

Probability that the employee will be greater than age of 44 = 100 - 84.1345= 15.86%

So the probability of number of employees between 38-44 years of age = P(X<44) - 0.5 = 84.1345 - 0.5 = 34.1345%

Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is **TRUE**.

1. Probability of employees less than age of 30 = P(X<30).Z = (30-38)/6

P(X ≤ 30) = P(Z ≤ (30-38)/6) = P(Z ≤ -1.333) = 9.12%

So the number of employees with probability 0.912 of them being under age 30 is

0.0912\*400 =36.48( or 36 employees).

Therefore the statement B is also **TRUE**.

1. **If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

Answer 03:

If X ∼N(μ1, σ1**2** ), and Y ∼N(μ2, σ2**2**) are two independent random variables then

X + Y ∼N(μ1 + μ2, σ1**2** + σ2**2**) , and X − Y ∼N(μ1 − μ2, σ1**2** + σ2**2**) .

If Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y ,

then Z ∼N(aμ1 + bμ2, a**2** σ1**2** + b**2** σ2**2**).

Therefore in the question 2X1~ N(2 u,4 σ**2**) and X1+X2 ~ N(μ + μ, σ**2** + σ**2**) ~ N(2 u, 2σ**2**)2X1-(X1+X2) = N( 4μ,6 σ**2**)

1. **Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**
2. **90.5, 105.9**
3. **80.2, 119.8**
4. **22, 78**
5. **48.5, 151.5**
6. **90.1, 109.9**

Answer 04:

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we can find it in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

We have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

We can calculate the X values. Z = (X-μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = XZ(-0.005)\*20+100

= -(-2.57)\*20+100

= 151.4Z(+0.005)\*20+100

= (-2.57)\*20+100

= 48.6

**Option D** is correct.

1. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**
2. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**
3. **Specify the 5th percentile of profit (in Rupees) for the company**
4. **Which of the two divisions has a larger probability of making a loss in a given year?**

